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T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	D	F4 ________________

2018
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Nowadays there is a global trend to promote the use of new energy automobiles, especially electric ones. We may have already seen electric vehicles running on the road, travelled by electric vehicle, or even owned one. However, the plans of development and relationships between the electric vehicles and charge stations vary from countries to countries and is crucial to further accelerate the switch to electric vehicles properly. In our paper, we aim to solve the above problems and to make out a realistic plan.

We build three models: **Functional Extreme Value Model, City-rural Developing χ Model and Vehicle-Charger Developing ODE Model.**

The first part of Functional Extreme Value Model is to rate different cities according to the population and area and fit it into continuous function, which has been used through the whole passages. Then we quantify the level of convenience, the difference between urban and suburban areas, the difference between two different types of charging. Thus we can get the Cost function scombining the level of convenience and cost of money . By calculating the extreme value of the cost function, we can get the distribution and total number of different chargers .

In City-rural Developing χ Model, we use the rating of cities or roads to rate the chargers in urban areas or rural areas. Next we can rank the stations in a sensible method and the rate decides the priority of them. Then we can χ define a benchmark that describe priority between city and rural.

To depict the relationship between time and density of stations , and to ascertain the priority of vehicles and chargers more precisely, we build the third model named Vehicle-Charger Developing ODE Model. Taking the social factors that influence the priority into account, we come up with two ordinary differential equations (ODE). Although solving the two equations may be kind of hard to us, we can use another way in which we use the stable characteristics of stationary and phase diagram to analyze the problem.

Though our models may have missed some factors like the climate, they are robust against changes of the various parameters that control the metrics, thus making it still an effective in planning the development of electrical vehicles and chargers.

Planning for Electric Vehicles: Our Approaches

Team #73937

February 13, 2018

Contents

I. Introduction

i. Background

Nowadays there is a global trend to promote the usage of new energy automobiles, especially electric ones. The promotion of electrical vehicles will bring about the Economic benefits. More significantly, They also typically generate less noise pollution than an internal combustion engine vehicle, whether at rest or in motion.

With the governments promoting the use of vehicles, we may have already seen electric vehicles running on the road, travelled by electric vehicle, or even owned one. However, as the the plans of development and relationships between the electric vehicles and charge stations vary from countries to countries and is crucial to further accelerate the switch to electric vehicles properly. In our paper, we aim to solve the above problems.

ii. Restatement of Problem

In the essence, the assigned five tasks put forward the following 3 issues.

1. The ideal number of charger stations and their distribution pattern among urban, suburban and rural areas.
2. The planning and investing method for the electric vehicles' development of a certain country.
3. The analysis and classification for countries of various factor properties.

To make it more specific,

- *Task 1* is specific as it raises the problem about a certain company, Tesla. It involves issue 1 and additionally classify the charging stations into two kinds: destination charging and supercharging; *Task 2* is consisted of three parts. In
- *Task 2a*, issue 1 is involved. In *Task 2b* and *Task 2c*, issue 2 is involved. The former one focuses more on the modes of development while the latter one focuses more on the course of development.
- *Task 3* and *Task 5* involves issue 3. *Task 5* requires a more general conclusion than *Task 3*.
- *Task 4* inspires us to take the factors we paid less attention to into consideration, thus making our solution more realistic.

iii. Concept Specification

1. **Urban area**: An urban area is a human settlement with high population density and infrastructure of built environment?and is considered to locate in the center of a city.
2. **Suburban area**: The prosperity degree of suburban area is second to urban areas and is considered to locate surround the urban areas.
3. **Rural area**: The total areas that strip out of urban areas and suburban areas.
4. **Roads**: A road is way connecting two cities which cars can drive on.

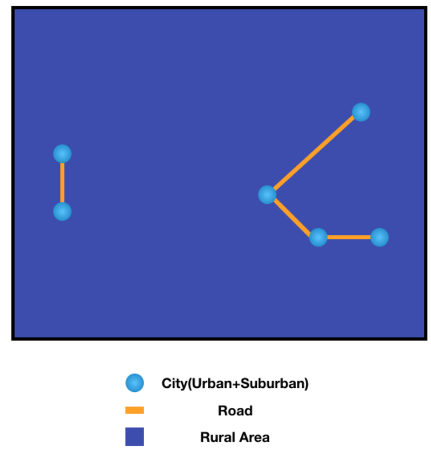

Figure 1: *City Distribution Assumption*

5. **Charging Stations**: A charging station is a place where people can charge their electric vehicles, but batter-swap is not involved. There might be several chargers in one station. Different companies have different kinds of charging stations. Take Tesla as an example, they offer mainly two types of charging stations: supercharging and destination charging.

iv. General Assumptions

1. Urban and suburban areas distribute in the pattern of *Concentric zone model*. When considering rural areas, we view urban and suburban circles as many particles in the map as is shown in Figure 1.
2. Cities can be classified into 10 standard levels based on the population and area of them, from small town to metropolis. Cities of same level share exactly same properties, while cities of different levels show differences in various aspects.
3. Densities of chargers and vehicles represent the distribution pattern better than absolute numbers.

4. Chargers in rural areas are meant for drives of long distance, usually between cities. Thus we assume them to be high-voltage, fast-charging stations distributed near roads.

II. Functional Extreme Value Model

i. Considerations

This model aims at discussing the optimal number, placement, and distribution of charging stations. We find it vital to consider many aspects. Respectively, they are:

1. **Charging station construction trade-off.** Denser distributions would result in higher costs which can be directly quantified by money, and meanwhile make it more convenient for electric vehicle owners, which cannot be directly quantified by money. Thus, it is important to characterize the perfectness of distribution.

2. **Differences between urban, suburban and rural areas.** In terms of charging station construction, different construction and maintenance (including ground rent) costs are the main factors.

3. **Reasonably take the geography factor into account.** That is, how we quantify the cities' and roads' distribution in map.

4. **Differences between different levels of cities.** For example, the final model's distribution must clearly reflect the difference in density of chargers between New York and Lynchburg (a small city in Virginia).

5. **Differences in density distribution between different types of chargers.** Specifically, Tesla Inc. currently offers two types of charging stations: destination charging and supercharging. We should take their different properties into account.

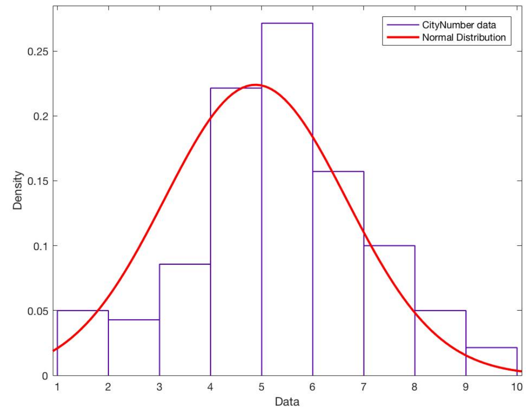

Figure 2: *City Level Classification*

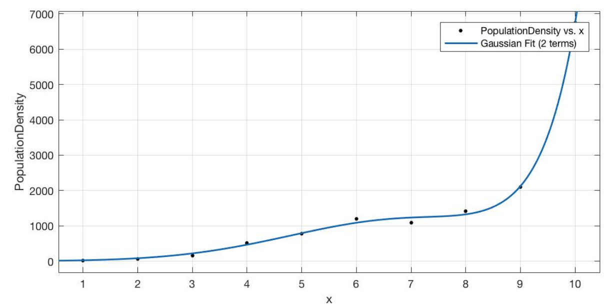

Figure 3: *Population Density Fitting*

ii. Specific Assumptions

1. According to the datasets which rank the population and scale of American cities(including their own suburban area), we select 150 major cities as representatives in our charger-construction planning process. Total population of these cites accounts for most of the U.S. population, so our assumption is rational.

2. We focus on the cities' coordinate data (longitude and latitude) to depict concentration and vehicle flow rates between cities.

iii. City Characterization

To make our model realistic, we discuss variation of density distribution of chargers in different city levels. Cities are classified into 10 standard classes using the formula:

$$CityRating = \ln \sqrt{Population * Area} \quad (1)$$

The rating result agrees with most mainstream rankings of U.S. cities pretty well. As the classification result shown in Figure 2, cities of level 4-7 take the most part. As discrete model would definitely result in excessive free parameters, we fit it into continuous Gaussian distribution function:

$$f(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-\mu)^2}{2\sigma^2}} \quad (2)$$

Using the same classifying standard we calculated the average population density distribution about city level and fit it into two-term Gaussian distribution function as shown in Figure 3.

$$\rho_p(r) = a_1 e^{-[\frac{x-b_1}{c_1}]^2} + a_2 e^{-[\frac{x-b_2}{c_2}]^2} \quad (3)$$

where σ, μ, a_1, a_2, b_1, b_2, b_1 are coefficients.

According to Tesla Inc., destination charging stations locate on hotels, shopping malls, restaurants in the cities. Supercharging stations, on the other hand, locate on central city and outer roads to provide essential support for road trips. We name the density of supercharging stalls in urban area as ρ_{lu}, the density of supercharging stalls in suburban area as ρ_{ls}, the density of destination stalls in urban area as ρ_{mu}, the density of destination stalls in suburban area as ρ_{ms} to reprensent the distribution of two types of charging stations within cities.

Then we filter the roads between cities to calculate the number of supercharging stations in the rural area (on the roads. To avoid repeat count, only cities with direct roads are taken into consideration. Based on the geographic information and city levels, we developed two rules:

Abbreviation	Description
r	Level of city
$f(r)$	Distribution of cities on levels
$A(r)$	Distribution of area on levels
$\rho_p(r)$	Distribution of population density on levels
S_r	Average area of cities on level r
N_r	Number area of cities on level r
c_l	Cost of a supercharging stall
c_m	Cost of a destination charging stall
ρ_{lu}	Urban supercharging stalls/City area
ρ_{ls}	Suburban supercharging stalls/City area
ρ_{mu}	Urban destination charging stalls/City area
ρ_{ms}	Suburban destination charging stalls/City area
l	Total length of candidate roads
x	Interval of supercharging stations on the road
$C_{Ru}(r)$	Rental of a stall in level r city urban area
$C_{Rs}(r)$	Rental of a stall in level r city suburban area
$Y_d(r)$	Depreciable life of a supercharging station
N_0	Total city number
q_l, q_m, K_{half}	Coefficients

Table 1: *Nomenclature of Functional Extreme Value Model*

- $Level_i + Level_j \geq 12$
- $Dist(i,j) \leq 400km$

The filtered 171 roads sums up to 41584.7 km in length (The list is in Appendix). They are viewed as supercharging station construction candidates. Due to the requirement of long road trips, supercharging stations are scattered isometrically on the roads. We assume the interval to be x, which is about 144km currently. In our further discussion, x is regarded as a variable.

iv. Evaluation Function

We use an evaluation function Z to quantify the best distribution in final condition (everyone switched to all-electric personal passenger vehicles) which reperesents the overall revenue of the society regard to a specific distribution $(\rho_{lu}, \rho_{ls}, \rho_{mu}, \rho_{ms}, x)$.

The evaluate function takes both money cost and non-monetary cost (or benefit) into account by reasonably quantification. Then the optimum decision problem can be seen as a functional extreme value problem. We assume density distribution function has good properties(quite smooth).

$$Z_{tot} = -Z_{cost} + Z_{benefit} \tag{4}$$

Z_{cost} consists of Z_C(construction cost) and Z_R(rental cost), measured by money directly. More specifically,

$$\begin{aligned} Z_C &= \sum_{r=1}^{10}[S_r N_r \sum_{i=l,m} c_i(\rho_{iu}(r) + \rho_{is}(r))] + \frac{l}{x}c_l \\ &= N_0 \int_1^{10} \sum_{i=l,m} c_i(\rho_{iu}(r) + \rho_{is}(r)) f(r) A(r) \mathrm{d}r + \frac{l}{x}c_l \end{aligned} \tag{5}$$

$$Z_R = \int_1^{10} \sum_{j=s,u} C_{Rj}(\rho_{lj}(r) + \rho_{mj}(r)) f(r) A(r) \mathrm{d}r \tag{6}$$

where $C_{Ru}(r)$ means rental cost for a supercharging stall in the urban area. As destination charging requires only a wall connector to be installed at restaurants and hotels, the rental are neglected. Considering the evolution of cities, we assume:

$$C_{Ru}(r) = (100 + \frac{400}{1 + e^{-x+5.5}}) * 12Y_d \tag{7}$$

$$C_{Rs}(r) = C_{Ru}(r-1) \tag{8}$$

Meanwhile,

$$\begin{aligned} Z_{benefit} = &\int_1^{10} \sum_{i=l,m} q_j(1 - e^{-\frac{\rho_{is}(r) ln2}{\rho_p(r) K_{half}}}) \rho_p(r) f(r) A(r) \mathrm{d}r \\ &+ \int_1^{10} \sum_{i=l,m} q_j(1 - e^{-\frac{\rho_{iu}(r) ln2}{\rho_p(r) K_{half}}}) \rho_p(r) f(r) A(r) \mathrm{d}r \\ &+ q_l e^{-rx} \end{aligned} \tag{9}$$

Abbreviation	Description
P_i	Rating of city i
P_i'	Rating of city i after iterations
R_{ij}	Rating of road connecting city i and j
$\rho(r)$	Charging station density of cities on level r
$f(r)$	Distribution of cities on levels
$A(r)$	Distribution of area on levels
$h(p)$	New density of cities
α, β	Coefficients

Table 2: *Nomenclature of City-Rural Developing χ Model*

v. Optimum Condition

Now we apply *Euler-Lagrange equation* to calculating the functional $\rho_{lu}(r), \rho_{ls}(r), \rho_{mu}(r), \rho_{ms}(r)$ and decide adequate value of x to maximize the evaluate function $Z_{tot} = -Z_{cost} + Z_{benefit}$. As for some constraint conditions for densities, we are considering using the *Lagrangian multiplier method* of the *variational method*.

The extreme value functions are:

$$\rho_{lu}(r) = \frac{\rho_p K_{half}}{ln2} ln \frac{q_l ln2}{c_l + c_{Ru}(r)} \tag{10}$$

$$\rho_{mu}(r) = \frac{\rho_p K_{half}}{ln2} ln \frac{q_m ln2}{c_m + c_{Ru}(r)} \tag{11}$$

$$\rho_{ls}(r) = \frac{\rho_p K_{half}}{ln2} ln \frac{q_l ln2}{c_l + c_{Rs}(r)} \tag{12}$$

$$\rho_{ms}(r) = \frac{\rho_p K_{half}}{ln2} ln \frac{q_m ln2}{c_m + c_{Rs}(r)} \tag{13}$$

III. City-rural Developing χ Model

i. Considerations

This model aims to:

- Ascertain the order to build chargers
- Figure out the investment in building chargers and the purchases in electric vehicles.

To reasonably realize the propose, we should follow the steps as follows:

- To distinguish the differences between stations in urban and rural areas by using two different methods to rate the two main types of stations.
- To ascertain the rational order to build chargers according to a certain mode (which will be explained further below).
- To ascertain the rational the sequential order of building chargers and buying cars.

ii. Specific Assumptions

1. For a certain city, the influence of the stations of nearby cities are so subtle that can be neglected.

2. The importance of the station within the city could be evaluated by the importance of the city.

iii. Station Rating

The base of the rating method is the classification of cities we use. For a certain charging station, there are two possibilities of its location: in the urban areas or in the rural areas. The method to rate these two kinds of station is sure to be different.

1. **Rate the stations in rural area.** The point of the station is the algebraic average of the point of two cities which are connected by the road that it's on.

$$R_{ij} = \frac{P_i + P_j}{2} \tag{14}$$

2. **Rate the stations in urban area.** A straightforward idea is to use the rating for the city as formula (1) . Considering the influence of cities nearby, we do an iteration to get a more precise rating.

$$P_i' = \frac{N_i P_i + \sum_{e_{i,j}=1} N_j P_j}{2N_i} \tag{15}$$

Considering the influence of stations within the same city, we add a penalty term into formula (15). The penalty term can be quantified as:

$$[e^{-\alpha\rho(r_i)A(r_i)} - 1]\beta \tag{16}$$

where α, β are coefficients.

iv. Building Priority

We draw a diagram to ascertain the order to build stations more intuitively. In the diagram, the x-coordinate is the point of stations in urban areas and the y-coordinate is the density of cities in terms of point(x-coordinate). Although the density of cities is now in terms of x(city level), we can make a easy transformation to get a new density of cities in terms of point.

Suppose the new density that is in term of p is $h(p)$. The sum of density of cities between r and $r + \mathrm{d}r$ is $f(r)\rho(r)A(r)\mathrm{d}r$. If we use the term p, the sum of density of cities between p and $p + \mathrm{d}p$ is $h(p)\mathrm{d}p$. It is obvious that the two consequences are equal. So we get the new density:

$$h(p) = f(r)\rho(r)A(r)\frac{\mathrm{d}r}{\mathrm{d}p} \tag{17}$$

Meanwhile, we draw a parallel axis which shows the distribution of the point of stations in rural areas as is shown in Figure (4).

The order to build stations can be concluded as: The higher the point is, the prior the stations to build. For example, we should first build the charger in city as red whose number is the area for <1>, then build the first-rank point on the axis.Then next build blue <2>, and followed by second-rank point on the axis.....

To quantify the overall city-rural developing priority for development of chargers, we must define an index to depict the difference between the average development order of city and rural chargers so that we can answer what the priority is in south Korea.

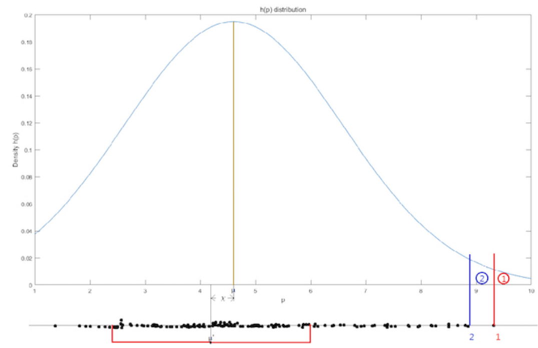

Figure 4: *$h(p)$ Distribution*

Abbreviation	Description
P_{veh}	Density of vehicles
P_{ch}	Density of chargers
$f(r)$	Distribution of cities on levels
V_d	Scrap rate of electric vehicles
GDP	GDP per capita (in dallors)
G	Gini coefficient
b_1, b_2, a_1, a_2, c	Coefficients

Table 3: *Nomenclature of Vehicle-Charger Development ODE Model*

Luckily, we find the continuous function h(p) conforms to normal distribution (requirement of *Central-limit Theorem*). We can fit it, get the parameter μ and σ and get a section $(\mu - \sigma, \mu + \sigma)$ where the area under this section takes 68.27%. And then we make a search in the number axis where discrete distribution of point in the road(rural) stand for construction priority. The concrete search method is that we use a section with length 2σ to include points in the number axis as many as possible. We find the proper position depicted in red color in Figure (4) above and we can derive the center point μ'.

Next,we define

$$\chi = \mu - \mu' \tag{18}$$

χ is actually a good index to describe priority between city and rural. If $\chi > 0$, we can roughly think city is prior and vice versa. In task3, we introduce more concrete function taking the gap of wealth and nation's economic condition into consideration to quantify this priority.

IV. Vehicle-Charger Developing ODE Model

i. Considerations

The densities of vehicles and chargers are sure to be closely related to time, meanwhile the densities of the two are interacted on each other. What's more, level of development and wealth distributions also influence the rate significantly, which are quantified by *GDP per capita* and *Gini coefficient*.

ii. Model Design

Take all these aspects into consideration, we give the following two ordinary differential equations (ODE):

$$\frac{d\rho_{veh}}{df} = a_1(-\rho_{veh} + K_{half}\rho_{ch}) + b_1\frac{GDP}{G} - V_d\rho_{veh} \tag{19}$$

$$\frac{d\rho_{ch}}{df} = a_2(\rho_{veh} - K_{half}\rho_{ch}) + b_2\frac{G}{GDP} + c * GDP \tag{20}$$

$\frac{GDP}{G}$ describes the ability of the consumption and the environmental awareness. Also, we roughly use GDP per capita to describe the government capacity to build chargers. Through we can get the function of density of electric vehicles and chargers, all in term of time.

By comparing the two graphics of the two functions, we can easily tell the order to build chargers and buy electric vehicles, thus problem 2b is solved. By using the function of density of electric vehicles, we can estimate how many years will be spent to

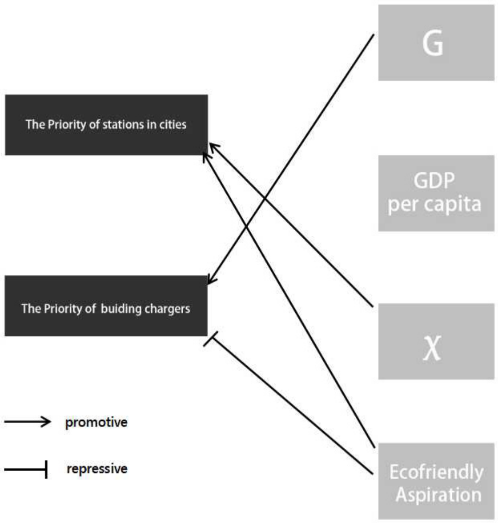

Figure 5: *The influence of social factors to the priority*

arrive a certain number of electric vehicles, thus problem 2c is solved.

But we need to find and adjust too many free parameters' value so that the arithmetic solution is not so convincing and realistic. However, this binary ODE set is a *plane autonomous system* and has lots of good qualities. Let the left side of this set be zero and we derive that the stationary solution uniquely exists and lies in first quartile in the ρ_{veh}-ρ_{ch} plane. What's perfectly good is that this stationary solution is always stable?and you can freely change any of the positive parameter in the ODE but the curve starting form(0,0) drawing near the stationary solution point is always in first quartile and approximates the stationary point alone one of two special directions.

Further more, we can use the ρ_{veh}-ρ_{ch} phase diagram to quantify the priority. First the stationary

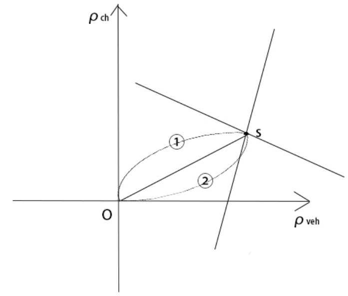

Figure 6: *Schematic Diagram*

solution is:

$$\rho_{veh} = \frac{b_1}{V_d} \cdot \frac{GDP}{G} + \frac{b_2}{V_d} \cdot \frac{G}{GDP} \cdot \frac{a_1}{a_2} \tag{21}$$

$$\rho_{ch} = \frac{b_1}{2} \cdot \frac{GDP}{G} + \frac{b_2}{2} \cdot \frac{G}{GDP} \cdot \frac{a_1}{a_2} \tag{22}$$

We can know the slope of the line linking original point and stationary point(S):

$$K_{OS} = \frac{\rho_{ch}}{\rho_{veh}} \tag{23}$$

- If the ρ_{veh}-ρ_{ch} curve evolves like curve 1 in the figure, we think the develop trace for charger and vehicle can be described as "charger preceding vehicle"
- If the ρ_{veh}-ρ_{ch} curve evolves like curve 2 in the figure, we think the develop trace for charger and vehicle can be described as "vehicle preceding charger"

Because of the ρ_{veh}-ρ_{ch} curve's approximation to the special directions, we only have to consider the slope of the tangent of the curve at (0,0). We define this slope as K_ρ:

$$K_\rho = \frac{\mathrm{d}\rho_{ch}}{\mathrm{d}\rho_{veh}} \,\Big|_{\rho_{ch}=\rho_{veh}=0} = \frac{\frac{b_2 G}{GDP} + c * GDP}{\frac{b_1 G}{GDP}} \tag{24}$$

By comparing the K_ρ and K_{OS}, we can make a quantification and classify three modes as follows:

1. $J > 5\%$, we describe the mode as "vehicle preceding charger"
2. $-5\% < J < 5\%$, we describe the mode as "mixed development"
3. $J < -5\%$, we describe the mode as "charger preceding vehicle"

Where

$$J = \frac{K_\rho - K_{OS}}{K_{OS}} * 100\% \tag{25}$$

J becomes our very index derived from *Vehicle-Charger Developing ODE Model* to quantify the priority and give a prediction. The threshold 5% is determined by vast statistical data for different countries in *Task 3*.

V. Results and Analytics

i. Task 1

Using *Functional Extreme Value Model*, we solved the distribution of charging stations as presented in formula (10) - formula (13). The result is plotted in Figure (7), from which we can clearly observe the phenomenon of counterurbanization in U.S. mid-level cities.

The total sum of charging stations can be further calculated using:

$$N = \int_1^{10} \rho(r) f(r) A(r) \mathrm{d}r \tag{26}$$

From which we predicted that if everyone switched to all-electric personal passenger vehicles in the US, 14589 destination charging stalls and 11869 supercharging stalls (about 2158 supercharging stations) are needed. Tesla is on its way to achieve this amount goal, having fulfilled about 20.6% of it.

ii. Task 2

ii.1 Task 2a

We choose South Korea.

Based on the *Functional Extreme Value Model*, we can

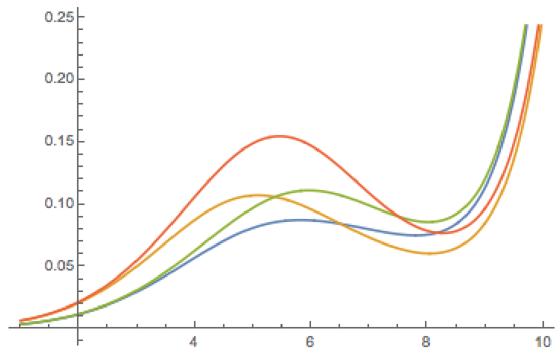

Figure 7: *$h(p)$ Curve of $\rho(r)$ under optimum condition*

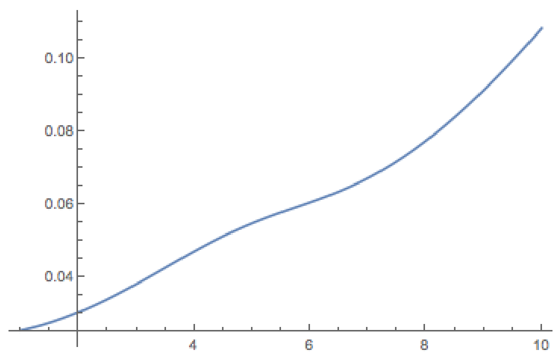

Figure 8: *The relationship between $\rho_u(r)$ and r in Korea*

apply a simplified one to Task 2a. We use $\rho_u(r)$ to denote the distribution of chargers in city (urban area) in terms of r.

The result can be shown as Figure (8).

- Total chargers' number in rural areas is: 46
- Total chargers' number in urban areas is: 139

The key factors that shaped the development of our plan are:

- The city's area and cities' concentration. They affect the number of chargers in rural areas because they influence the total length of the road.
- The density of people and the scale of cities. To some degree, it's a standard to measure urbanization. It affect the tendency of $\rho_u(r)$. South Korea is a developed country and the curve $\rho_u(r)$ is much like America.

ii.2 Task 2b

We apply the *City-Rural Developing χ Model* to the statistics of South Korea.

- For South Korea, the index J is 5.1 (>5). So vehicle is obviously prior than chargers.
- As for city-rural development priority of chargers, we can estimate from χ. For South Korea, μ=4.61, μ'=4.23, χ=μ-μ'=0.38>0, So city prior.

In emphTask 3, we will take wealth distribution and amount into consideration. For South Korea, there are deeper discussion in *Task 3*. Here we make a provisional recognition that the factor is only χ, which can describe a country's priority for city-rural problem on average as described in *City-Rural Developing χ Model*.

ii.3 Task 2c

Although we can?t present the analytical solution, We can adopt a way to estimate the rate of convergence from original point to stationary point.

But when we focus on the concrete evolution curve as a function of time(year).We must give reasonable constraint to the coefficient. If we can reduce some of the free coefficients, we may give a reasonable solution.

We assume that when in final condition, two ρ can have a fixed ratio for a maximized and comfortable convenience. Using the conclusion in *Task 2a*, we roughly see the charger?s average density as total number (derived in *Task 2a*) divided by nations area. And it?s exactly equal to our stationary solution. And we get a constraint of parameters by linking ODE model with *Task 2a*s results which is derived by *Task 1*?s model. Using this method we get a system of two element equations and we can express a_1 and a_2 by b_1, $b2$, V_d. So we reduce our free parameters.

For the arithmetic solution, we could take reasonable parameters and give the density evolving figure.

iii. Task 3

We have 2 questions to consider and quantify:

- How to select proper indexes to distinguish different countries? geographies, population density distributions and wealthy distributions which influence countries? rational developing trace.
- How to quantify the indexes? combination effect and specify the classification using phase diagram.

We make two assumptions:

- We assume that there are some main indexes to determine the trace for growing the charging network to simplify the vast and confusing characters between countries.They are: Gini Coefficient (G), GDP per capita (GDP), χ and eco-friendly aspiration of people (Eco).
- We can adopt our ODE model?s criteria for classification and results from rating methods in *City-rural Developing χ Model* directly, and take every index above into consideration.

We combined two models above to solve *Task 3*:

ODE model classifies the vehicle-charger developing priority.

Different countries have different G and GDP, so the developing priority can be different according to our criteria in ODE stationary solution?s converging curve model presented in *Task 2b*. There are 3 types, and we can easily classify different countries.

City-rural Developing χ Model predicts city-rural developing priority.

Here we think the classification for priority is determined by χ and Eco. Eco can be described as $lg(\frac{GDP}{G})$. Because $\frac{GDP}{G}$ describe the eco-friendly aspiration. When this term increase, people have more money and as a group, they has bigger purchasing power. So people from city want to improve cities? environment quality and chargers in city is prior to that in rural areas. So we write our combined index M as:

$$M = \chi * lg(\frac{GDP}{G}) \quad (27)$$

Notice that in vehicle-charger developing model we define Eco effect as $\frac{GDP}{G}$.But we want to decrease this item?s effect in city-rural developing model, so we take the logarithm. Further more, we define the sector:(-∞,0),[0,1],(1,∞) respectively refers to rural prior, mix and city prior when considering M.

We refer to the G&GDP data from UN (2017) and we get the indices of these countries. We can gain our χ from the geography of countries. The index M and the results are shown in Table (4) (some entries lack data)

We find that some of countries are special so that our model is not a perfect classification.The countries selected have some unique properties but some of the countries fits in our models :

- **Australia**: main cities concentrate in a little part of nation and going from east to west, crossing the whole needs chargers which may not lie close to any cities.We can see Australia as an extreme nation for nonuniform city distribution in land.
- **China**: The unique properties of China is not so notable so China fits in our models.
- **Indonesia**: Many islands decrease cities? road relatedness and may increase charger density in urban area. We can see Indonesia as a typical nation for weaker city relevance as the world?s biggest Archipelagic State.
- **Saudi Arabia**: The oil resources in Saudi Arabia are so plenty that it becomes a forceful resistance to promote electrical vehicles and chargers. Also, the area of deserts makes up nearly half of the total area of land, so the relatedness between cities are weakened, which is another unique properties.
- **Singapore**: The total area of Singapore is as small as a city, so there is no need to take rural areas and the stations on the road into consideration.

We list some interesting cities above. Actually we deal with many other countries and we find the distribution of χ and J can all be fitted by *Gaussian distribution*. According to *Central-limit Theorem*, our method for quantification of the classification is roughly reasonable. The concrete threshold :(0&1 for M and 5% for J)

iv. Task 4

In our analysis, we consider the three following key factors that all have close relationships with technology:

1. Total types of transportation options
2. The development of share services and public transportation
3. Charging mode and the quality of battery

Factor (1) and Factor (2) are expected to slower our step to promoting the use of electric vehicles, while Factor (3) are expected to accelerate the use of electric vehicles.

First, with increasing types of transportation options, people have more and more alternatives which are as convenient as cars – or even more convenient than cars. A few people who originally choose the electric cars may change their idea.

Second, the development level of share services is gradually increasing, which means more and more people will use the same vehicles, so the electric cars per capita will increase at a much slower rate or even stop. There are some similarity in share services and public transportation. If the use of vehicles increase, the road must be more crowded; At the same time, the public transportation is becoming more and more convenient and comfort. So the loss of people who use electric vehicles are inevitable.

Countries	GDP(dollar)	G%	J%	Prior1	Chi	lg(GDP/G)	M	Prior2
Australia	56135	30.3	11.2	vehicle	-0.14	5.27	-0.73	rural
China	8583	46.5	1.3	mix	0.44	4.27	1.86	city
Indonesia	3858	36.8	-5.9	charger	?	4.03	?	?
Saudi Arabia	20029	45.9	4.7	mix	?	4.63	?	?
Singapore	53880	45.8	8.8	vehicle	0.31	5.07	1.56	city
South Africa	6089	62.5	-6.4	charger	0.31	3.99	1.29	city
Finland	45693	21.5	14.9	vehicle	0.09	5.32	0.48	mix
Colombia	7720	53.5	-2.1	mix	?	4.16	?	?
Japan	38550	37.9	6.9	vehicle	0.28	5.01	1.4	city
South Korea	29730	34.1	5.1	vehicle	0.38	3.94	1.49	city

Table 4: *Some countries' priority of vehicle-charger and city-rural*

Third, the charge mode of electric vehicles will directly influence the promotion of electric vehicles. Nowadays most charge modes, regardless of super-charge or determination charge, all need some time to get enough energy. The cost of time is a restrict factor that make a few people hesitate to buy a electric vehicle. But if enough rapid battery-swap stations are built, a part of people are expected to buy electric vehicles with the huge decrease in time cost of charging. In addition, the improvement of battery will increase the electric powered run time. In this way, the time cost will also be lessened and more people will use electric vehicles.

v. Future Work

We have three big models using 3 different methods to build and solve: functional extreme value problem, graph theory to abstract geography factors and ODE and its correlate plane autonomous system?s analysis. We can complete our model by:

1. add constraints which can be written as the Lagrangian multiplier of the variational method. The constraints come from grid power supply limit and the like...But it?s hard for us to gain these data.
2. We can make the country's geography factors more clear by in detail defining and calculating "interactions" between city and rural areas.
3. Pitifully we don't get a timeline for concrete time nodes because of a lack of decision in some of ODE parameter value. But actually time evolution tendency is clear and we can assume we know t(50%) so we can write down the time for t(10%) t(30%) t(100%).

Actually our models are correlated to each other by some parameters and show great covariation when the city we study changes. We hope the prediction can be more comprehensive in the future.

VI. References

1. The World Factbook – Central Intelligence Agency (www.cia.gov/library/publications/the-world-factbook/)
2. List_of_United_States_urban_areas - Wikipedia (www.wikiwand.com/en/List_of_United_States_urban_areas)
3. FindUs - Tesla (https://www.tesla.com/findus#/)

4. Supercharging - Tesla
(https://www.tesla.com/support/supercharging)

Handout (intended for task 5)

To: All leaders attending the submit
Subject: Make a national plan to migrate personal transportation towards all-electric cars
To make a rational plan, it is vital for us to consider the following key factors that play a dominant role in your plan.

- GDP and Gini coefficient: If your countries have high GDP and high Gini coefficient, you should first make investment in building chargers ,and the achievement of your plan will take more time. Else, you should pay more attention on electric cars.
- The Allocation of Resources: If the oil resources is plenty enough to run all the motor vehicles for a long time, which is a forceful restriction of electric vehicles' development, the achievement of your plan will take more time. Else it will be easier to promote the electric vehicles in your nations.
- Concentration of Cities: If the level of concentration of cities is low(e.g. the areas of water and the desert takes a relatively big percentage in total areas), you'd better first make investment in chargers in rural areas. In another word, the construction of stations on the road is more important, thus the road construction is fundamental to the achievement of your plan.
- Ecofriendly Aspiration of People: If people in your nations have high ecofriendly aspiration, the achievement of your national plan will be more smoothly and cost less time. Else, rising the ecofriendly aspiration is of great importance for the realization of your plan.
In addition to the factors listed above, there remains a lot of factors due to some unique characteristics of your nations and should be considered as well.

Taking all these key factors into account, you can make the rough plan out by combining the current development of electric vehicles of your country.